Collimated transmission calculations

\[ T = \exp(-\epsilon^* CL) = 0.980 \quad \text{at } \lambda = 632.8\, \text{nm} \]

\[ \epsilon^* = -\ln(T) / CL \]

\[ = \frac{-\ln(0.980)}{\left(\frac{2 \% \text{l lipid}}{4000}\right)(1 \, \text{cm})} = 40 \left[ \frac{\text{cm}^{-1}}{\% \text{l lipid}} \right] \]

\[ \mu_s = (40)(2 \% \text{l lipid}) = 80 \, \text{cm}^{-1} \]
Integrating sphere

- An integrating sphere in essence is an enclosure to contain and diffuse input light so that it is evenly spread over the entire surface area of the sphere. This diffusion is completed through two mechanisms:
  - a Lambertian reflectance surface (or coating) and
  - a Lambertian reflectance surface is a physical ideal – 100% reflectance and completely uniform angular spreading of the light energy on the first bounce.
  - a geometrical sphere shape.
- The intensity from the incident radiation $I_0$ varies only as the viewing angle of the surface $\theta$. When this ideal Lambertian surface is combined with a spherical enclosure the geometry of the sphere ensures that every point within a sphere receives the same intensity or light as every other part of the sphere at the first bounce.

Lambertian source

A consequence of Lambert's cosine law is that when a Lambertian surface is viewed from any angle, it has the same apparent radiance.

\[
\text{Flux (from } dP_1 \text{ to } dP_2) = \frac{\cos \theta_1 \cos \theta_2}{\pi S^2} dP_2 = \frac{\cos \theta_1 \cos \theta_2}{\pi (2R \cos \theta_1)(2R \cos \theta_2)} dP_2 = \frac{dP_2}{4\pi R^2} = \frac{dP_2}{A_{\text{surface area}}}
\]
\[ \mu_{\text{eff}} = \sqrt{3 \mu_a (\mu_a + \mu_s')} \approx \sqrt{3 \mu_a \mu_s'} = 44.6 \text{ cm}^{-1} \]

\[ \mu_a \mu_s' = \frac{44.6^2}{3} = 663 \text{ cm}^{-2} \]
Reflection calculations

Reflected power = 2 µW

Reflected power = 1.58 µW

100% reflectance standard

Reflected power

$R_\infty \approx \exp(-\mu_a 7\delta) = \exp(-\frac{7}{\sqrt{3(1 + \frac{\mu'_s}{\mu_a})}})$

$\mu'_s = \frac{49}{3(-\ln(R_\infty))^2} - 1 = 300$

Reflection calculations

$\mu'_s = \frac{633 \text{ cm}^{-2}}{\mu_a}$ from integrating sphere

$\mu'_s = 300 \mu_a$ from paper reflectance

$\mu_a^2 = \frac{633}{300} = 2.21 \text{ cm}^{-2}$

$\mu_a = 1.48 \text{ cm}^{-1}$

$\mu'_s = 446 \text{ cm}^{-1}$

A fundamental question

Consider a cup of coffee with cream. Shine a laser onto liquid.

The reflectance is $R$.

Now, add some water to the coffee/cream.

Does the $R$ increase, decrease, or remain the same?
Diffuse reflectance, $R_d$

\[ T = e^{-\mu_a L} = e^{-\frac{\mu_a n}{\mu_s}} \]

Step size $= 1/\mu_s$

Total photon path $= L \approx n/\mu_s$

Diffuse reflectance, $R_d$

Add water...

\[ f = \frac{V + \Delta V}{V} \]

Step size $= 1/(\mu_s f)$

n steps before escape out surface

Total photon path $= L \approx n/(\mu_s f)$

Diffuse reflectance, $R_d$

\[ T = e^{-\mu_a L} \]

Diffuse reflectance, $R_d$

Total diffuse reflectance, $R_d$

\[ R = e^{-\mu_a L} \]

\[ \mu_a L = -\log_e(R) \]

Contributions of various absorbing molecules
Total diffuse reflectance, $R_d$

$$R \approx e^{-\mu_a L}$$

$$R_2 \approx e^{-(\mu_a + \Delta \mu_a) L}$$

$$L = \frac{-\ln \left( \frac{R_2}{R} \right)}{\Delta \mu_a}$$

Reflectance Reduced scattering/absorption

$$R_d = \exp(-\mu_a L_{\text{eff}})$$

$$R_d \approx \exp(-\mu_a 7\delta)$$
Reflectance as a function of source detector distance

\[ R(r) \]
Reflectance as a function of source-detector separation
Test material:
Intralipid phantom
Biological tissue

\[ \frac{M_{\text{total}}}{M_{\text{total, std}}} \rightarrow R \]

\[ M(r) \rightarrow \delta \]

\[ \delta \approx -1/\text{slope} \]
Added absorber method

![Diagram showing R_total and M_total](image1)

![Graph showing R_total vs. R_total + added absorber](image2)

Added absorber method

![Diagram showing R_total and M_total](image3)

![Graph showing R_total vs. R_total + added absorber](image4)
Added absorber method

Optical penetration depth, $\delta$

$$T = k \times \exp(-r / \delta)$$

$$\delta = \sqrt{\frac{D}{\mu_a}} = \frac{1}{\sqrt{3\mu_a(\mu_a + \mu_s')}}$$

$$= \frac{1}{\mu_{\text{eff}}}$$

Calculations

$$R = 0.855$$

$$R = 0.496$$

milk + 0.100 ml of ink into 400 ml milk
($\mu_a\text{ ink stock} = 2480 \text{ cm}^{-1}$)

$\mu_a\text{ ink} = 2480/4000 = 0.62 \text{ cm}^{-1}$
Calculations

\[ R_\infty = \exp(-\mu_a \, 7\delta) = \exp\left(-\frac{7}{\sqrt{3(1 + \frac{\mu_s}{\mu_a})}} \right) \]

\[ \frac{\mu_s}{\mu_a} = \frac{49}{3(-\ln(R_\infty))^2} - 1 = 665 \]

\[ \frac{\mu_s}{\mu_a + \mu_{a\text{ ink}}} = \frac{49}{3(-\ln(R_\infty))^2} - 1 = 32 \]

cont

\[ \mu_a \times 665 = (\mu_a + \mu_{a\text{ ink}}) \times 32 \]

\[ \mu_a = \frac{665 \mu_{a\text{ ink}}}{1 - \frac{32}{665}} = 0.031 \text{ cm}^{-1} \text{ for milk (2\% lipid) at 633nm} \]

\[ \mu_{a\text{ ink}} = 0.62 \text{ cm}^{-1} \]

CCD camera with tilted source

**Diagram:**

- Negative point source
- 3D cos \(\alpha_t\) + 2AD
- Positive point source
- 3D cos \(\alpha_t\) - 2AD
- Air
- Phantom
- Light
- Optical fibre
- CCD camera
- \(\Delta x\)
- \(\rho_1\)
- \(\Delta y\)
- \(r\)
- \(\Delta z = 3D \cos \alpha_t\)
\[ \delta = \frac{1}{\sqrt{3}} \left( \mu_a + \mu_s \right) \]

\[ mfp' = \frac{1}{\mu_a + \mu_s} \]

\[ \mu_a = \frac{\delta^2}{3} mfp' \]

\[ \mu_s' = \frac{1}{mfp'} - \mu_a \]