Exam in Medical Image Analysis - TBMI02

<table>
<thead>
<tr>
<th>Description</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examination date:</td>
<td>2015-01-17</td>
</tr>
<tr>
<td>Room:</td>
<td>KÅRA</td>
</tr>
<tr>
<td>Time:</td>
<td>08.00-12.00</td>
</tr>
<tr>
<td>Course code:</td>
<td>TBMI02</td>
</tr>
<tr>
<td>Exam code:</td>
<td>TEN2</td>
</tr>
<tr>
<td>Course name:</td>
<td>Medical Image Analysis</td>
</tr>
<tr>
<td>Department:</td>
<td>Department of Biomedical Engineering</td>
</tr>
<tr>
<td>Number of questions in the exam:</td>
<td>16</td>
</tr>
<tr>
<td>Number of pages:</td>
<td>9</td>
</tr>
<tr>
<td>Responsible teacher during the exam:</td>
<td>Hans Knutsson</td>
</tr>
<tr>
<td>Phone number during the exam:</td>
<td>013-286727, 070-3262694</td>
</tr>
<tr>
<td>The teacher visits the exam room:</td>
<td>One visit between 10 and 11</td>
</tr>
<tr>
<td>Course administrator:</td>
<td>Karoline Waltelius 013-286738</td>
</tr>
<tr>
<td></td>
<td><a href="mailto:karoline.waltelius@liu.se">karoline.waltelius@liu.se</a></td>
</tr>
<tr>
<td>Permitted equipment:</td>
<td>Calculator with empty memory, Tefyma, Beta, Physics handbook, dictionary</td>
</tr>
</tbody>
</table>

The aim of the assignments is to test knowledge and understanding of central concepts in the course. The exam consists of two parts:

Part 1. Consists of twelve assignments. The full answer should be written in the space after the question. Calculations or other information not specifically asked for should NOT be given! Maximally two points per assignment.

Part 2. Consists of four assignments. Complete solutions should be given on separate papers. Maximally four points per assignment.

To pass the exam you need 20 points. Remember to hand in both the exam and the answers to part 2. Unreadable writing will render zero points!

Good luck!

AID-number: .........................
Part 1

1. The k-space samples corresponding to the left image is illustrated in k-space sampling 1 below. Which of the k-space samplings below is most likely to have been used in the middle image and in the right image? The origin is located in the center for all sampling patterns.

![K-space samples](image)

k-space 1  k-space 2  k-space 3

k-space 4  k-space 5  k-space 6

k-space sampling number _____ was used for the middle image.

k-space sampling number _____ was used for the right image.
2. The function \( f(x) = 1 + \frac{1}{4}x^4 \) shall in a least square sense be approximated as well as possible with the help of the two basis functions \( b_1(x) = 2^x \) and \( b_2(x) = 2^{-x} \). The approximation is calculated from the values in the sample points \( x \in \{-1, 0, 1\} \). The middle point has half the weight of the other two. Give the coefficients for \( b_1 \) and \( b_2 \) that gives the best approximation.

\[
\begin{align*}
\quad b_1 &= \quad \quad b_2 = \quad \quad \\
\end{align*}
\]

3. Kalle is trying to register two images using a phase based registration algorithm. The original distance between the object in image 1 and image 2 is approximately 25 pixels. Kalle uses two quadrature filters with a center frequency of \( \frac{\pi}{4} \) and an appropriate spatial size. Unfortunately the two images are not at all aligned after the registration, despite the fact that Kalle tried to use 200 iterations.

a) Explain to Kalle why the registration does not work. (1 p)

b) Propose an efficient solution to the problem. (1 p)
4. State if the following assertions are true or false. Three correct answers give 1 point, four correct answers give 2 points.

<table>
<thead>
<tr>
<th>Statement</th>
<th>True</th>
<th>False</th>
</tr>
</thead>
<tbody>
<tr>
<td>Watershed is a border based segmentation method.</td>
<td>□</td>
<td>□</td>
</tr>
<tr>
<td>It is impossible to perform segmentation by using a registration algorithm.</td>
<td>□</td>
<td>□</td>
</tr>
<tr>
<td>To avoid clustering in the active contours algorithm, decreasing the number of nodes is a good idea.</td>
<td>□</td>
<td>□</td>
</tr>
<tr>
<td>It is hard to change the topology of an active contour.</td>
<td>□</td>
<td>□</td>
</tr>
</tbody>
</table>

5. In the kernel optimizer `krnopt` we prefer a uniform sampling in $N$ points over the Fourier domain $[-\pi, \pi]$ that is symmetric for positive and negative frequencies and has one sample in the origin.

a) Which sample points should be used for a 1D filter when $N = 3$? (1p)

b) Give the corresponding expression for a 1D sample pattern when $N$ is an arbitrary odd number. (1p)
6. In image registration based on optical flow, the error to be minimized can for example be written as

$$
\epsilon^2 = \sum_i (\nabla I_i(x_i)^TB(x_i)p - \Delta I(x_i))^2
$$

where \(\nabla I\) is the gradient of the image, \(B\) is a base matrix, \(p\) is the parameter vector to optimize and \(\Delta I\) is the intensity difference between the two images. Explain the mathematical/computational advantage of using an \(L_2\) norm for the error (compared to any other norm, e.g. the \(L_1\) norm where the error is simply the sum of the absolute values \((\epsilon = \sum_i |a_i - b_i|)\)).

7. Göran is analyzing some fMRI data, for each voxel he performs a statistical analysis to see if that part of the brain was active during the experiment. Since Göran is using a complicated algorithm to analyze the data it takes a lot of time to go through all the voxels. Göran tells you that voxels outside the brain are not interesting, can you help him to speedup the analysis? A slice of his fMRI data is given below.
8. You are looking at the plots of an optimized quadrature filter. What properties should you look for in the Fourier domain and in the Spatial domain when you judge the result of the optimization from these plots?

Fourier domain:

Spatial domain:

9. Decide if the following statements are true or false. Three right answers gives 1p, four right answers gives 2p.

<table>
<thead>
<tr>
<th></th>
<th>True</th>
<th>False</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Amplification of noise induced structures are mainly due to errors in the ( m )-func mapping.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. The ( \mu )-function is high in areas that just contain noise.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. The LP-filter in the adaptive filtering is not used when ( |C| &gt; 0 ).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. In 2D the local tensor and the GOP-representation contain the same information.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
10. Gunnar has collected some fMRI data using an MR scanner. Unfortunately the subject Linda tells Gunnar that she moved her head during the scan. Can you think of a way to improve the fMRI data before the statistical analysis?

11. Decide if the following statements are true or false for adaptive filtering. Three right answers gives 1p, four right answers gives 2p.

<table>
<thead>
<tr>
<th>Statement</th>
<th>True</th>
<th>False</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. The $\gamma_1$-image has low magnitude in areas that just contain noise.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. The $\mu_2$ mapping is independent of $|T|$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. If $|C| = 0$ only the lowpass filter is used.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. If the SNR in the image is increased the noise threshold in the $m$-function should be decreased.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

12. a) Two fundamental steps during an MRI are listed below. Describe the effect each step has on the spins of the hydrogen atoms in the examined object. (1p)

- Put the object in a spatially constant 3 Tesla magnetic field.
- Send a strong electromagnetic pulse of 127.7 Mhz into the object.

b) Given the conditions in a) what will happen if we add a gradient, $G_x$, orthogonal to the stationary magnetic field? (1p)
13. In MRI the protons are repeatedly excited with an RF pulse and it is common to excite the protons once for every line in k-space.

a) From a contrast perspective, explain why it is not a good idea to start the sampling directly after the RF pulse. (1 p)

A k-space sampling is done by using the gradient sequence given below, \((G_x(t), G_y(t))\). Only one excitation is used and the sampling is started at the origin.

\[
G_x^1 = 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 0, 0, 1, 1, 0, 0, 0 \\
G_y^1 = 1, 1, 1, 1, 1, 0, 0, -1, -1, -1, 1, 1, 1, 0, 0, -1, -1, -1, 0, 0, 1, 1, \\
G_x^2 = 0, 0, 1, 1, 1, 1, 1, -3, 0, 3, 1, 1, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0 \\
G_y^2 = 1, 1, 0, 0, 0, 0, -1, -1, -1, -1, 0, 0, 1, 1, 1, 1, 0, 0, -1, -1, -1, -1
\]

b) Give the formulas for how to calculate the time varying sample position in k-space \((k_x(t), k_y(t))\) from the gradients \((G_x(t), G_y(t))\). (1 p)

c) Draw the resulting scan pattern in k-space, first use \(G_x^1, G_y^1\) and then continue with \(G_x^2, G_y^2\) (without restarting from the origin). Neglect the constant \(\frac{\gamma}{2\pi}\) and use \(\Delta k_x = \Delta k_y = 1\). (1 p)

d) Explain how the EPI sampling method works for obtaining MR images. Also state one advantage and one disadvantage with EPI sampling compared to conventional sampling in MRI. (1 p)

14. The optical flow equation states that

\[
\nabla I^T v - \Delta I = 0
\]

where \(I\) is the image intensity.

a) State the two assumptions required for optical flow and use them to derive the optical flow equation. (2 p)

b) Why is it important that the parameter vector is updated in each iteration of the registration algorithm, instead of modifying one of the images? (1 p)

c) Why would a registration algorithm that is based on local phase be better than an algorithm that is based on the image intensity? (1 p)
15. We would like to use adaptive filtering on wave patterns on different wavelengths of similar type as in the test image used in the course. We use quadrature filters with a 6dB relative bandwidth of one octave, $\beta = 1$.

a) What range of wavelengths (in pixels) can we expect to detect by the quadrature filters if the center frequency $u_0 = \pi/3$? (1p)

b) What range of wavelengths (in pixels) can we expect to detect by the quadrature filters if the center frequency $u_0 = \pi/6$? (1p)

c) The local tensor representation is computed from five quadrature filters evenly distributed in one half plane in the Fourier domain and normalized such that $\|T\| \in [0, 1]$. For the tuning of the adaptive filtering we choose three tensors from different parts of the image.

\[
T_1 = \frac{2}{\sqrt{5}} \begin{pmatrix} 1 & 0 \\ 0 & 1/2 \end{pmatrix} \quad T_2 = \frac{3}{10\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad T_3 = \frac{3}{\sqrt{17}} \begin{pmatrix} 1/4 & 0 \\ 0 & 1 \end{pmatrix}
\]

We want the corresponding control tensors $C_1$, $C_2$ and $C_3$ to correspond to the local adaptive FD representation of the filter shapes in the figure below. The adaptive filters are of the same type as the filters used in the course. Define an $m$-function and a $\mu$-function that fulfills these requirements (2p).

![Filter shapes](image)

16. A tensor, $T$, is given by:

\[
T = \begin{pmatrix} 5 & -1 \\ -1 & 5 \end{pmatrix}
\]

a) Compute the Frobenius norm $\|T\|$ and compute the double argument vector representation, $z$, (the GOP-representation) of this tensor. (1p).

b) Compute the tensors $T_1$ and $T_2$ such that

\[
T = T_1 + T_2
\]

where both $T_1$ and $T_2$ are rank 1 tensors. Finally compute the corresponding double argument vector representations, $z_1$, $z_2$, $\|T_1\|$ and $\|T_2\|$. What observations can be made? (3p)