Exam in Medical Image Analysis - TBMI02

Time: 2008-01-07 kl 14.00-18.00
Place: KÅRA
Responsible teacher: Hans Knutsson, tel 0703262694
Aids: Calculator with empty memory, Tefyma, Beta, Physics handbook

The exam consists of two parts:

Part 1 Consists of twelve assignments. The aim of the assignments is to test the common knowledge and understanding of central concepts in the course. The answers shall shortly describe your solution and preferably be written in the space after the question. Maximally two points per assignment.

Part 2 Consists of four assignments. Complete solutions shall be given on separate paper. Maximally four points per assignment.

To pass the exam you need 20 points.

Good luck!

Code: .................................
Part 1

1. Explain why it is not a good idea to represent the orientation of a line in a 2D plane using the angle between that line and a reference line (e.g. the x-axis).

2. The output signal from a quadrature filter in a certain neighborhood has a phase of $\pi/2$ and an amplitude of 2. For a different neighborhood the same quadrature filter gives an output signal having a phase of $-\pi/4$ and a magnitude of $\sqrt{2}$. If you add the two neighborhoods, what will the output from the same quadrature filter be in the new neighborhood.
3. What is the purpose of using spatial weight and spatial ideal functions in the filter optimization? Motivate the shape of the spatial ideal and spatial weight functions.

4. Sketch how the frequency weight should be adapted for different signal to noise ratios in the images. Motivate the difference between the curves.
5. The task is to approximate the function \( f(x) = 2 \cos^2\left(\frac{\pi}{4}x\right) \) as well as possible in the least square sense using the two basis functions \( b_1(x) = 1 \) och \( b_2(x) = x \). The fit is computed using the values in the sample points \( x \in \{-1, 0, 1\} \). Give the coefficients for \( b_1 \) and \( b_2 \) that yields the optimal fit. The point \( x = 0 \) has twice the weight of the other two.

6. Name one edge based and one region based segmentation method and explain how they work.
7. The gray values of the image (three copies) below is given by: \( \text{im} = 0.5 + x \) inside the circle, \( \text{im} = 0.5 - y \) inside the ring, and \( \text{im} = 0.6 + \sqrt{x^2 + y^2} \) outside the ring. The origin is located in the middle of the image. Draw the segmentation results when:

a. the image is thresholded at 0.5.

b. using a watershed algorithm that is started at low points.

c. a snake algorithm starting outside the ring.

(The image is also an example of that the visual impression of an image may differ from what objective measurements indicate - don’t let this disturb you ;-) )

8. Decide if the following statements are true or false. Three right answers gives 1p, four right answers gives 2p.

<table>
<thead>
<tr>
<th>Statement</th>
<th>True</th>
<th>False</th>
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<tbody>
<tr>
<td>1. The ( \mu )-function is low in areas that just contain noise.</td>
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<td>2. ( \gamma_2 ) depends on both the magnitude and the shape of the signal.</td>
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<tr>
<td>3. The control tensor ( \mathbf{C} ) and the local tensor ( \mathbf{T} ) have identical eigenvectors.</td>
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<td></td>
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<tr>
<td>4. The ( m )-function is a mapping of (</td>
<td>\mathbf{T}</td>
<td>).</td>
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</tbody>
</table>
9. Below $T_1$ and $T_2$ are plotted. For $T_1 \lambda_1 = 5$ and $\lambda_2 = 1$. For $T_2 \lambda_1 = 2$. Estimate $e_1$ and $e_2$ for both tensors and compute $T_1$ and $T_2$.

10. In MRI you sometimes want to get signals only from a thin slice of a body. This can be accomplished by selectively exciting this slice. Explain how this is done.
11. The figures below show five resulting enhancement filters in the Fourier domain. Combine each filter with the corresponding filter parameters $\gamma_1$ och $\mu_2$. Three right answers gives 1p, five right answers gives 2p.

Filter 1

Filter 2

Filter 3

Filter 4

Filter 5

$\gamma_1 = 0.5$ och $\mu_2 = 0$ corresponds to filter ______
$\gamma_1 = 1.0$ och $\mu_2 = 0$ corresponds to filter ______
$\gamma_1 = 0$ och $\mu_2 = 0.5$ corresponds to filter ______
$\gamma_1 = 0.5$ och $\mu_2 = 1$ corresponds to filter ______
$\gamma_1 = 1$ och $\mu_2 = 1$ corresponds to filter ______
12. In 3D the shape of a local structure tensor of rank 1 corresponds to a discus, a rank 2 tensor corresponds to a cigar shape and a rank 3 tensor to a sphere. What is the fundamental shape of the 3D tensor if the local 3D image consists of:

a) a 3D plane

b) a 3D line

c) noise
Part 2

13. One way of making a fast MR scan is to collect k-space data using a spiral sequence, see figure 13.

The k-space trajectory is in polar coordinates given by:

\[ \rho = \alpha t \]
\[ \varphi = \beta t \]
\[ t_{n+1} = t_n + \Delta t \]

a) Give the x- and y-gradients as a function of time, \( t \) [S]. (2p)
b) The samples are taken every 10th \( \mu \text{S} \) (\( \Delta t = 10^{-5} \)), \( \alpha \) has been chosen so that the sampling density in the radial direction is high enough not to cause aliasing (ghosting). Find the value of \( \beta \) that gives just high enough sample density also in the angular direction (the out-most part of k-space being critical) to reconstruct an image of size 800x800 pixels without aliasing. (2p)
14. Registration of one image, $I_2$, to another, $I_1$, can be described by the following three equations:

\[
\begin{align*}
I_2(x + v(x)) & \approx I_1(x) \\
I(x + v(x)) & \approx I(x) + v(x)T \nabla I(x) \\
v(x) & \approx Bp
\end{align*}
\]

a). Describe in words the meaning of each of the three equations and explain why it is necessary to iterate to find the correct $v(x)$? (2p)

b) Write down the system of equations for finding the best $p$ for one iteration. What is the minimum number of pixels required to find a unique solution if $B$ corresponds to an affine transformation in 2D? (2p).
15. In adaptive filtering we have a particular interest for line structures 1.5-8.5 pixels wide.

a) What bandwidth (in octaves) is appropriate for the quadrature filters in the estimation the local structure of the image? (1p).

b) Motivate with reference to the center frequency a suitable spatial support size for the quadrature filters (1p).

c) The local tensor representation is computed and normalized such that $|\mathbf{T}| \in [0, 1]$. For the tuning of the adaptive filtering we choose three tensors from different part of the image.

\[
\mathbf{T}_1 = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \quad \mathbf{T}_2 = \frac{1}{10 \sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \mathbf{T}_3 = \frac{2}{\sqrt{10}} \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{3} \end{pmatrix}
\]

We want the corresponding control tensors $\mathbf{C}_1$, $\mathbf{C}_2$ and $\mathbf{C}_3$ to correspond to the local adaptive filter in the figure below. Define a $m$-function and a $\mu$-function that fulfills these requirements (2p).
16. A complex valued double argument or gop representation of orientation in 2D can be computed from the tensor components as:

\[ T = \begin{pmatrix} t_1 & t_2 \\ t_2 & t_3 \end{pmatrix} \]

\[ z(\phi) = t_1 - t_3 + i 2t_2 \]

Compute the magnitude of the double argument representation \(|z|\) for the tensor

\[ T = \lambda_1 e_1 e_1^T + \lambda_2 e_2 e_2^T \]

as a function of \(\lambda_1\) and \(\lambda_2\).

Hint: First compute the double argument representation of

\[ T_1 = \lambda_1 e_1 e_1^T \quad \text{and} \quad T_2 = \lambda_2 e_2 e_2^T \]

where \(e_1 = [\cos(\phi), \sin(\phi)]^T\) and \(e_2 = [\sin(\phi), -\cos(\phi)]^T\).